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Frequency Modification Using Newton's Method and Inverse Iteration Eigenvector Updating

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An iterative procedure is proposed for determining structural changes necessary for modifying natural frequencies of a structure in a prescribed manner. The procedure uses a Newton's method approach to determine a structural change that satisfies the equation of motion of the modified structure. The Newton equation is the same as that derived from sensitivity analysis and from linear perturbation methods. Iterative application of this equation can be used to predict accurately the structural change for either large or small frequency modifications. Inverse iteration is used to update the structural eigenvector between Newton iterations. The application of this procedure to a finite element analysis is described. Natural frequency modification of a finite element structural model is shown to converge rapidly to accurate results.

Nomenclature

A = matrix of frequency constraint coefficients
b = vector of prescribed frequency shifts
c_{ij} = participation of jth baseline mode to changes in the ith modified mode
c_i = vector containing participation of all baseline modes to changes in the ith modified mode
f_i = ith natural frequency of the baseline structure
f_i^* = prescribed value of the ith natural frequency
f_i' = ith natural frequency of the modified structure
g = functional containing frequency constraints and the objective function
i = index associated with the ith mode
j = index associated with the jth mode
K = stiffness matrix of baseline structure
K' = stiffness matrix of modified structure
k_e = element stiffness matrix
l_1 = the first mode number with a prescribed frequency
l_n = the nth mode number with a prescribed frequency
M = mass matrix of baseline structure
M' = mass matrix of modified structure
M_i = generalized mass associated with the ith baseline mode
m = total number of design variables used for modification
m_e = element mass matrix
n = total number of frequency constraint equations
p_r = value of the rth structural property variable
p_r' = value of the rth property in the modified structure
$[\ ]^T$ = transpose of the preceding matrix or vector
α = vector of fractional property changes
α_r = fractional change to rth property
γ_i = ith un-normalized mode shape of modified structure
ΔK = change to stiffness matrix
ΔM = change to mass matrix
Δp_r = change to the rth structural property
$\Delta\lambda_i$ = change to ith eigenvalue of the baseline structure
$\Delta\psi_i$ = change to ith mode shape
$\Delta\psi_{ih}$ = change to ψ_{ih}
δ = error in satisfying frequency constraints
λ_i = ith eigenvalue of the baseline structure
λ_i^* = prescribed value of the ith eigenvalue
μ = penalty parameter
Ψ = matrix of mode shape vectors of the baseline structure
ψ_i = ith mode shape of the baseline structure
ψ_i' = ith mode shape of the modified structure
ψ_{ih} = hth entry of the ith mode shape vector
$\|\ \|_2$ = Euclidian norm of the enclosed vector

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Background

In the vibration design of a structure, it may be desirable to modify the natural frequencies and mode shapes of an existing model in a prescribed manner in order to obtain a satisfactory dynamic response. Appropriate structural changes must then be determined to meet the design objectives. Inverse modification problems are concerned with finding these structural changes in some analytical way.

With the advent of finite element analysis, perturbation methods have become popular for inverse modification problems. The philosophy of perturbation methods is to investigate the solution of a modified structure by considering the modification as a perturbation of the linear baseline system.

Stetson[1-3] used a linearized perturbation equation in which all nonlinear incremental terms were discarded. In addition, the changes in the structural mode shapes were described by a linear combination of the baseline modes. This allows first-order approximate solutions to be found for frequency and mode shape modification problems.

Various refinements of the approach have been made since the original work of Stetson.[4-8] Hoff et al.[5] presented a two-stage, predictor–corrector method for frequency modification. In this method, an additional perturbation analysis, based on the linear analysis, is performed to improve the first-order estimate of the structural changes. It has been found by Welch[9] that the predictor–corrector scheme is adequate for problems with linear property changes, but is unable to compute nonlinear property changes accurately.

Kim et al.[10] have proposed a method in which the nonlinear equations of the modified structure are solved using mathematical programming. Minimum weight solutions are found with the aid of a starting vector.

More general inverse eigenvalue problems have been studied by mathematicians.[11-15] Inverse eigenvalue problems involve the specification of one or more eigenvalues of a matrix and the evaluation of how the coefficients of the matrix need to change to meet the prescribed eigenvalues. Friedland et al.[14] gives a review of various applications of inverse eigenvalue problems and describes four general methods for solving them. All of these use a Newton's method approach in which approximate solutions are computed iteratively until they converge to the correct result. Three different algorithms are used to update the eigenvectors: 1) recalculating with an eigenvalue analysis; 2) updating with inverse iteration; and 3) updating by evaluating the participation of the baseline vectors in the modified vector. This last method has been used in perturbation methods[2-6], but the present authors have found that large-mode shape changes are not predicted accurately. This is because the participation factors are computed from a lineralized equation, which is only accurate for small changes.

Introduction

In this study, a Newton's method algorithm is proposed for frequency modification of finite element models. This work is based on research previously done on inverse eigenvalue problems of a general mathematical nature.[11-15] It is shown that the basic equations are the same as those used in linearized perturbation analysis[2] and sensitivity analysis[16] to predict small structural changes. In perturbation analysis, the lineralized equations have also been used as the first stage in a predictor–corrector analysis. To compute large structural charges accurately, numerous iterations of the predictor–corrector algorithm may be needed, as was demonstrated in the recent work by Bernitsas and Kang.[7]

In this study, it is shown that by using the linear perturbation equation in Newton's method application, accurate results for large or small structural changes can be obtained. To accomplish this, the structural eigenvectors of the prescribed modes must be updated accurately between iterations. An inverse iteration scheme is used as an accurate eigenvector updating technique that is not too computationally expensive.

In the first section, general expressions for the change in the frequency and mode shape are derived by considering the variation of a single structural property. Following that, the algorithm used to solve inverse modification problems is discussed. In the second section, the application of the frequency modification algorithm to the finite element program INSTRUM is discussed. In the third section, some examples are presented showing the frequency modification of a structural finite element model. The results obtained from Newton's method iteration are verified by a finite element reanalysis of the modified structure.

Derivation of the Frequency Modification Equations

In the dynamic analysis of a structure modeled with finite elements, the natural modes are computed with the eigenvalue equation:

$$[K - \lambda_i M]\psi_i = 0 \tag{1}$$

where λ_i and ψ_i are defined as the ith baseline eigenvalue and eigenvector of the system. In frequency modification problems, the objective is to find the properties of a modified structure, as represented by K' and M', such that

$$[K' - \lambda_i^* M']\psi_i' = 0 \tag{2}$$

where λ_i^* is a prescribed eigenvalue. There are as many Eqs. (2) as there are prescribed eigenvalues. A necessary component of solving this problem is finding the modified eigenvector ψ_i'.

To determine K' and M' in the above equations, it is necessary to find how λ_i and ψ_i change with respect to variations in the properties of the structure. If K and M are constructed from a finite element model, the properties of the structure will consist of the properties of its constituent elements. These properties might include element thickness, cross-sectional area or density. Considering λ_i and ψ_i to be functions of a number of structural properties p_r, and considering λ_i to be distinct, the following differential relations can be derived:[16]

$$\psi_i^T\left[\frac{\partial K}{\partial p_r} - \lambda_i \frac{\partial M}{\partial p_r}\right]\psi_i = M_i \frac{\partial \lambda_i}{\partial p_r} \tag{3}$$

$$\psi_j^T\left[\frac{\partial K}{\partial p_r} - \lambda_i \frac{\partial M}{\partial p_r}\right]\psi_i = -\psi_j^T[K - \lambda_i M] \frac{\partial \psi_i}{\partial p_r} \quad j \neq i \tag{4}$$

where $M_i = \psi_i^T M \psi_i$, the ith generalized mass.

If many p_rs change, then the incremental changes on λ_i and ψ_i are given by

$$\Delta\lambda_i = \frac{\partial \lambda_i}{\partial p_1}\Delta p_1 + \ldots + \frac{\partial \lambda_i}{\partial p_m}\Delta p_m \tag{5}$$

$$\Delta\psi_i = \frac{\partial \psi_i}{\partial p_1}\Delta p_1 + \ldots + \frac{\partial \psi_i}{\partial p_m}\Delta p_m \tag{6}$$

where m is the total number of properties that may change. The partial derivatives in Eqs. (5) and (6) are in general not constants. By allowing finite variations of p_r and by approximating numerically the partial derivatives at the baseline values of p_r, Eqs. (5) and (6) give linear approximations to the frequency and mode shape changes incurred for a finite structural modification.

Substituting the partial derivatives from Eqs. (3) and (4) into Eqs. (5) and (6) with the appropriate indices gives

$$\Delta\lambda_i = \sum_{r=1}^{m} \frac{1}{M_i}\left[\psi_i^T \frac{\partial K}{\partial p_r}\psi_i - \lambda_i \psi_i^T \frac{\partial M}{\partial p_r}\psi_i\right]\Delta p_r \tag{7}$$

$$\psi_j^T(\lambda_i M - K)\Delta\psi_i = \sum_{r=1}^{m} \psi_j^T\left[\frac{\partial K}{\partial p_r} - \lambda_i \frac{\partial M}{\partial p_r}\right]\psi_i \Delta p_r \quad j \neq i \tag{8}$$

Equation (7) may be used for frequency modification problems by substituting the appropriate frequency change $\Delta\lambda_i$ and solving for Δp_r.

Equation (7) provides a linear approximation to the changes required to produce the prescribed frequency shift. Stetson and Palma[2] derived similar equations by means of a perturbation analysis in which all incremental terms of order higher than one were deleted. By repeatedly solving Eq. (7) in a Newton's method algorithm, the results can be made to converge to a design solution that which satisfies Eq. (2). Convergence can be achieved in general, if the initial guess provided by the baseline quantities is reasonably close to the design solution and that the nonlinearity of modified equation is not too severe. It is also necessary that after each iteration the mode shapes, frequencies, and structural properties are accurately updated.

There are three methods for updating the mode shapes between iterations. One is to perform a free vibration analysis on the modified structure as was done with the baseline structure. This is an accurate but costly method. The two other approaches make use of the baseline modes to predict the modified mode shapes. One method, which has been used in perturbation methods, involves describing the ith mode shape change as a linear combination of all mode shapes excluding the ith:[4]

$$\Delta\psi_i = \Psi c_i \tag{9}$$

where $c_{ii} = 0$ and Ψ is a rectangular matrix containing the baseline mode shapes as columns. The assumption that $c_{ii} = 0$ is made for the sake of convenience. Because only $n - 1$ elements of $\Delta\psi_i$ may be specified uniquely, only $n - 1$ elements of c_i can be determined uniquely leaving one entry to be set arbitrarily. One such arbitrary choice is to set $c_{ii} = 0$, which means that the modified mode shape ψ_i' will have a unit value participation from its baseline counterpart ψ_i.

Applying this to Eq. (8) gives

$$(\lambda_i - \lambda_j)M_j c_{ij} = \sum_{r=1}^{m} \psi_j^T \left[\frac{\partial K}{\partial p_r} - \lambda_i \frac{\partial M}{\partial p_r}\right] \psi_i \Delta p_r \quad j \neq i \tag{10}$$

Substituting Eq. (10) into Eq. (9) gives the equation

$$\Delta\psi_{ih} = \sum_{k \neq i} \frac{\psi_{kh}}{(\lambda_i - \lambda_k)M_k} \sum_{r=1}^{m} \psi_k^T \left[\frac{\partial K}{\partial p_r} - \lambda_i \frac{\partial M}{\partial p_r}\right] \psi_i \Delta p_r \tag{11}$$

Equation (11) may be used to compute the eigenvector change resulting from some structural change Δp_r. An approximation is introduced when a truncated set of baseline modes, rather than a full set, is used in Eq. (9). In most practical problems, this is unavoidable and requires the use of engineering judgment in choosing the number of baseline modes to be used.

The major drawback of this method is that Eq. (11) is not able to calculate accurately large eigenvector changes. This is because it is only a linear approximation to the actual eigenvector change, so it is only reliable for small changes. When this equation is derived using perturbation methods, the nonlinear incremental terms have to be neglected.[4] Some of these terms contain $\Delta\psi_i$, thus if large eigenvector changes are involved, these nonlinear terms may not actually be small and their neglect is unjustified. It is important to update accurately eigenvectors otherwise Newton's method will converge to an incorrect solution.

A more accurate method for updating eigenvectors is inverse iteration, which is described in Ref. 17. Using this approach with Newton's method has been suggested by Friedland et al.[14] To compute the modified mode shape resulting from a structural change, the following equations are solved:

$$[K + \Delta K - \lambda_i^*(M + \Delta M)]\gamma_i = \psi_i \tag{12}$$

$$\psi_i' = \frac{\gamma_i}{\|\gamma_i\|_2} \tag{13}$$

where

$$\Delta K = \sum_r \frac{\partial K}{\partial p_r} \Delta p_r \tag{14}$$

$$\Delta M = \sum_r \frac{\partial M}{\partial p_r} \Delta p_r \tag{15}$$

Determining the modified mode shape involves the decomposition of the matrix in brackets in Eq. (12). Computationally, this is more efficient than performing an finite element re-analysis. This procedure has been incorporated into a finite element program called INSTRUM (Inverse Structural Modification). For every frequency that is to be modified, the corresponding mode shape is updated between iterations; the other mode shapes do not need to be updated.

Finite Element Implementation of Newton's Method

To apply Eq. (7) to an inverse modification procedure in a finite element program, the quantities $\partial K/\partial p_r$ and $\partial M/\partial p_r$ must be calculated. The properties p_r are really properties of the individual elements, not properties of the structure as a whole. The structural stiffness and mass matrices are summations of the element stiffness and mass matrices:

$$K = \sum_e k_e \tag{16}$$

$$M = \sum_e m_e \tag{17}$$

Derivatives of the structural matrices are, therefore, summations of derivatives of the element matrices:

$$\frac{\partial K}{\partial p_r} = \sum_e \frac{\partial k_e}{\partial p_r} \tag{18}$$

$$\frac{\partial M}{\partial p_r} = \sum_e \frac{\partial m_e}{\partial p_r} \tag{19}$$

The expressions (18) and (19) are substituted into Eqs. (7) and (8).

If $\partial k_e/\partial p_r$ is a constant, then p_r is called a linear property change. If $\partial k_e/\partial p_r$ is a function of p_r, then it is a nonlinear property change. An example of a linear property change is the cross-sectional area of a bar element, the axial stiffness of a bar being directly proportional to the area. An example of a nonlinear property change is the thickness of a plate element, the stiffness being proportional to the cube of the thickness. Properties of the element mass matrix are linear.

The design variable α_r is defined as the fractional change of p_r; i.e.

$$\Delta p_r = \alpha_r p_r \tag{20}$$

This equation is substituted into Eqs. (7) and (8) and by defining

$$k_{er} = \frac{\partial k_e}{\partial p_r} p_r \tag{21}$$

$$m_{er} = \frac{\partial m_e}{\partial p_r} p_r \tag{22}$$

the resulting frequency modification equation is

$$\Delta\lambda_i = \sum_{r=1}^{m} \sum_e \frac{1}{M_i} [\psi_i^T k_{er} \psi_i - \lambda_i \psi_i^T m_{er} \psi_i] \alpha_r \tag{23}$$

This equation is now suitable for use in a finite element program.

Equation (23) has been implemented in the program INSTRUM, which is designed to solve frequency modification problems. To solve this equation for the design variables α_r, the number of design variables must be greater than or equal to the number of prescribed frequencies. If m is equal to n, then one may look for a unique solution satisfying Eq. (23). If m is greater than this, there may be an infinite number of solutions which satisfy Eq. (23). If this is the case, a solution to Eq. (23) may be found by recasting it as a minimization problem.

Suppose that the set of Eq. (23) for $i = l_1, \ldots, l_n$ is written in the compact form

$$A\alpha = b \tag{24}$$

where $b_i = \Delta\lambda_i$ and

$$A_{ij} = \sum_e \frac{1}{M_i} [\psi_i^T k_{ej} \psi_i - \lambda_i \psi_i^T m_{ej} \psi_i] \tag{25}$$

Define the vector δ as the error in Eq. (24):

$$\delta = A\alpha - b \tag{26}$$

For the case $m > n$, a minimum change solution satisfying $\delta = 0$ can be found by minimizing the functional:

$$g(\alpha) = \sum_r \alpha_r^2 + \mu\delta^T\delta \tag{27}$$

The first term in $g(\alpha)$ is a measure of the design change. A minimum weight objective might be used instead of a minimum change to produce a modified structure with the prescribed frequencies and the least possible weight. It has been shown in Ref. 8 that this type of objective often leads to pathological solutions and that better results are usually obtained when minimum change solutions are looked for. In this study, a minimum change objective is always used.

The second term in Eq. (27) penalizes the functional for not satisfying the constraint equations. The penalty parameter μ may be adjusted to emphasize either the satisfaction of the frequency constraints or minimization of the objective function. A large μ achieves the former, a small μ the latter. It was found that adequate results could be obtained by using the following scaling factor:

$$\mu = \frac{m}{|\Sigma_k A_{hk}|} \tag{28}$$

A strict application of Newton's method can be used to minimize $g(\alpha)$ but it is computationally more efficient to use a quasi-Newton procedure using a weak line search and BFGS update. A description of this algorithm can be found in Ref. 18. In addition to the terms shown in Eq. (27), penalty terms are included in $g(\alpha)$ to steer the design variables away from the region of $\alpha_r < -1$, where property changes are physically unrealizable.

The conditions under which repeated solutions of the linear equations will converge to an acceptable design solution for the modified structure are dependent upon the actual case being solved. In general, the process converges provided the initial guess (i.e. the baseline) is reasonably close to the solution and if the modified equation is not too nonlinear. This means that the procedure is more likely to converge for a small frequency change than for a large one. However, nothing is guaranteed.

It is found that faster convergence with Newton's method is achieved if λ_i is replaced with λ_i^* on the right-hand side of Eq. (23). Inverse iteration is used to update ψ_i between iterations and the Rayleigh quotient is used to update the eigenvalues. Also, property variables p_r are updated between iterations using

$$p_r' = (1 + \alpha_r)p_r \tag{29}$$

If some of the properties are nonlinear, then $\partial k_e/\partial p_r$ must be recalculated at the current value of p_r', otherwise a false convergence may result.

Examples

The structural model considered represents the frame of a mast tower. The model is composed of regular six degrees of freedom per node beam elements, which, in this case, have axisymmetric properties. The design variables of interest are the natural frequencies of the tower in different modes and examples are presented to illustrate the effectiveness of INSTRUM in analyzing the problem. A baseline analysis of the mast structure shows that the third mode, for example, is a twisting mode with a frequency of 7.84 Hz, as shown in Fig. 1.

In the two cases presented, the frequency of the third mode is to be raised by 10, 30, 50, 100, and 200%. Thus, the number of frequency constraints n is one. The structural elements that are to be changed are identified in Fig. 2 by nine element groups. Each group forms a beam comprised of three elements.

In each of the two cases, the property change variables are used and so $m = 3$. As m exceeds n, an infinite number of design solutions may satisfy the frequency constraint and so a minimum change solution is sought, as described in the last section.

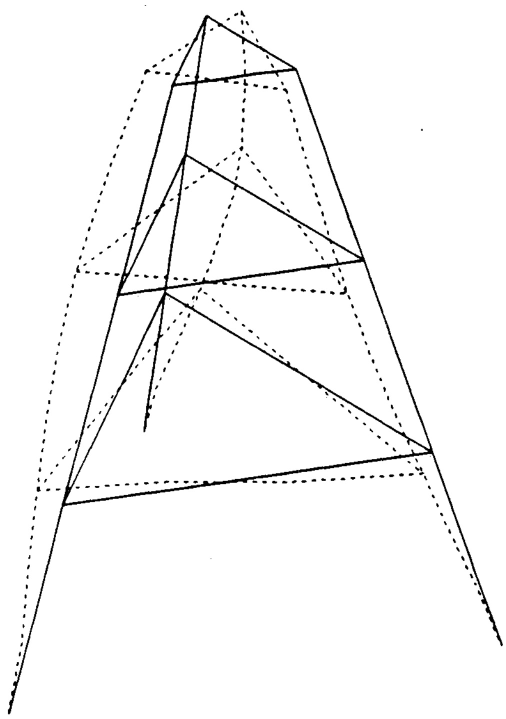

Fig. 1 Twisting mode of a mast structure, f_3 = 7.84 Hz.

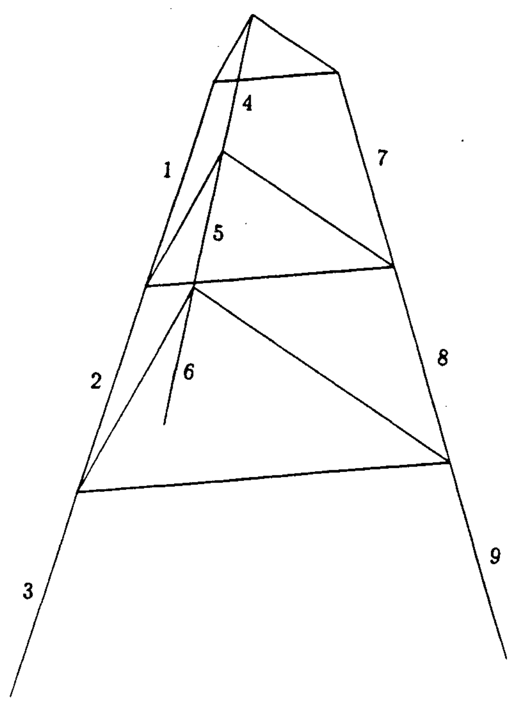

Fig. 2 Finite element model with nine element groups. Each group contains three beam elements.

Case 1

The element sets corresponding to the three design variables are shown in the note to Table 1. The nine element groups have been arranged vertically into three element sets.

It may be seen from Fig. 1 that the strain energy of the twisting mode is stored primarily in the bending of the columns. The cross beams are rotated in a rigid-body fashion as a result of this bending, but they experience very little elastic

Table 1 Element sets[a] and corresponding INSTRUM results for Case 1

f_3^*/f_3	No. iterations	α_1	α_2	α_3	$\Sigma_r \alpha_r^2$	f_1'/f_1	f_2'/f_2	f_3'/f_3
1.10	3	0.468	0.470	0.470	0.661	1.04	1.04	1.10
1.30	4	1.80	1.80	1.80	9.75	1.13	1.13	1.30
1.50	4	3.65	3.66	3.66	40.1	1.23	1.23	1.50
2.00	5	10.9	11.5	11.1	372	1.46	1.47	2.00
3.00	9	52.4	56.9	53.7	8860	1.93	1.94	3.00

[a]Set no.	Variable	Element groups
1	α_1	1, 2, 3
2	α_2	4, 5, 6
3	α_3	7, 8, 9

Table 2 Element sets[a] and corresponding INSTRUM results for Case 2

f_3^*/f_3	No. iterations	α_1	α_2	α_3	$\Sigma_r \alpha_r^2$	f_1'/f_1	f_2'/f_2	f_3'/f_3
1.10	3	0.059	0.330	0.635	0.516	1.04	1.04	1.10
1.30	4	0.247	1.36	2.87	10.1	1.14	1.14	1.30
1.50	4	0.535	2.71	6.90	55.2	1.26	1.26	1.50
2.00	5	1.50	5.90	28.9	871	1.57	1.57	2.00
3.00	9	14.9	17.8	92.8	9140	2.04	2.04	3.00

[a]Set no.	Variable	Element groups
1	α_1	1, 4, 7
2	α_2	2, 5, 8
3	α_3	3, 6, 9

deformation themselves. Therefore, one possible way of increasing the frequency of the mast in this mode is to increase the second moment of area of the column sections. Thus, in this case, each of the three property variables α_1, α_2, and α_3 are prescribed as fractional changes of the second moment of area of their associated element groups. In the case of the beam element, the mass is independent of the second moment of area and, thus, the results obtained assume no change in the mass as a result of the stiffness changes. If different elements are used, the effect of mass changes is automatically incorporated.

Table 1 shows the results for five frequency modification analyses. The first column contains the frequency objectives. The second column contains the number of Newton iterations required for convergence. The last three columns contain the results of a finite element re-analysis using the computed design change. In all cases, the re-analysis shows that the modified structure meets the frequency objective to within a negligible error. Also note that the frequencies of the first and second modes have increased, but by a smaller proportion than the third frequency. If these frequencies needed to be controlled during the modification, then additional frequency constraints would have to be added to the analysis. As it is, the first and second frequencies are repeated roots and so the frequency modification equations cannot be applied. This difficulty can be avoided by adding a small asymmetric modification to the baseline structure that separates these frequencies but that has an otherwise insignificant influence on the structure's dynamics.

The number of iterations required in each analysis is dependent upon the size of the prescribed frequency shift. It can be seen that as size of the change becomes larger, more iterations are required for convergence, as one would expect from a Newton's method algorithm.

Note that the second moment of area of each element set is changed by nearly the same amount in each analysis and so the symmetry of the original structure is preserved in the modified structure. Thus, the mode shape of the twisting mode of the modified structure will be very similar to that of the original structure.

Case 2

For this case, the nine element groups were arranged horizontally into three element sets, as shown in the note to Table 2.

The property to be changed is the second moment of area of the beam elements. The results obtained from INSTRUM are shown in Table 2. The re-analysis of the modified structure shows that the frequency constraint is satisfied in each analysis. Also, the number of iterations required for convergence is similar to what was required in case 1.

The design solutions are quite different from those obtained in case 1. Note that the largest fractional change is to be made to the elements closest to the base of the mast. This is because in the twisting mode, the strain energy is greatest where the curvature due to bending is the greatest—near the base. The smallest fractional change is to be made to the elements farthest from the base. The curvature in these elements is small, so changing their moment of inertia will have little effect on the natural frequency. The eigenvector for the modified structure for a 200% frequency increase is depicted in Fig. 3.

The total fractional change, given in column six of the results, is larger for case 2 than it is for case 1, with the exception of $f_3^*/f_3 = 1.10$, when case 1 is larger. This means that for small frequency changes, the program predicts that the horizontal grouping is more efficient, while for large changes, the vertical grouping is better. One might wish to find a more efficient design by specifying a total of nine design variables, so that the second moment of area of each element set can change independently. The results for nine independent variables are shown in Table 3. Observe that the design solutions are very similar to those found in Table 2 except that the total change is about three times as large because there are now three times as many variables. Also note that overall minimum change solutions have not been found, as some of the results in Table 1, in fact, have a smaller total change. The reason for this is that the method determines a minimum change solution to the current constraint equations in each iteration of the algorithm. However, the accumulation of these minimum change solutions does not necessarily lead to an overall minimum change solution.

Summary

1) A method is proposed for modifying an existing structural model in order to change one or more of its natural frequencies by a prescribed amount. The method uses linearized frequency modification equations in a Newton's method algorithm. The frequency modification equations are similar to those derived from sensitivity or perturbation analysis. One-

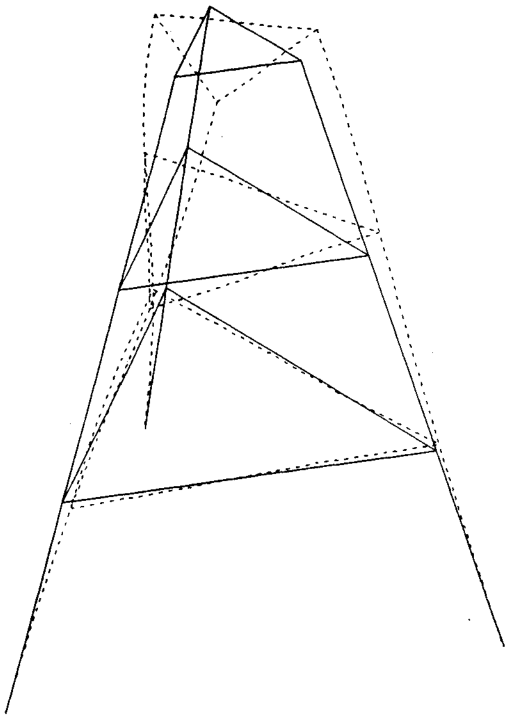

Fig. 3 Modified eigenvector for f_3^*/f_3 = 3.00.

Table 3 INSTRUM results for nine independent property changes

f_3^*/f_3	1.10	1.30	1.50	2.00
α_1	0.070	0.242	0.507	1.36
α_2	0.333	1.34	2.58	5.17
α_3	0.629	2.89	7.04	31.0
α_4	0.070	0.242	0.509	1.37
α_5	0.334	1.34	2.58	5.47
α_6	0.631	2.89	7.05	30.7
α_7	0.070	0.242	0.509	1.32
α_8	0.334	1.34	2.59	5.02
α_9	0.630	2.89	7.06	30.1
$\Sigma\alpha_r^2$	1.54	30.6	169.9	2900
No. iterations	6	10	11	11
f_3'/f_3	1.10	1.30	1.50	2.00

step inverse iteration is used to update structural eigenvectors between Newton iterations. An eigenvalue analysis of the baseline system is required initially, but no subsequent eigenvalue analyses need to be performed.

2) The method is implemented in a finite element program INSTRUM with applications for frequency modification problems. For underconstrained problems, where the number of frequency constraints is fewer than the number of design variables, minimum change solutions are sought with the aid of a quasi-Newton optimization routine. The method may be used to determine accurate design solutions for either linear or nonlinear property changes.

3) Examples presented for modifying the third natural frequency of a mast structure show that accurate solutions can be obtained for frequency shifts of 10–200%. The satisfaction of the frequency constraints was verified with a finite element re-analysis of the modified structure.

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